Get to Work, Mix It Up, Go the Distance, and Lower the Cost!

8.4

Using Rational Equations to Solve Real-World Problems

LEARNING GOALS

In this lesson, you will:

- Use rational equations to model and solve work problems.
- Use rational equations to model and solve mixture problems.
- Use rational equations to model and solve distance problems.
- Use rational equations to model and solve cost problems.

If you haven't noticed, learning is not limited to the classroom, but can be part of everyday life. In fact, there's the old adage that confirms this: You learn something new everyday.

But learning alone does not equate to success. You need to apply that knowledge and gain experience. In many ways, the classroom sets the foundation for success, but by applying your knowledge, you can shine!

Mathematics is no different. When you gain a deep understanding of the mathematics you use in the classroom and can relate it to real-world situations, you are able to do your own problem-solving and gain success.

How have you used mathematics in your everyday life? How do you think you will use math in the future?

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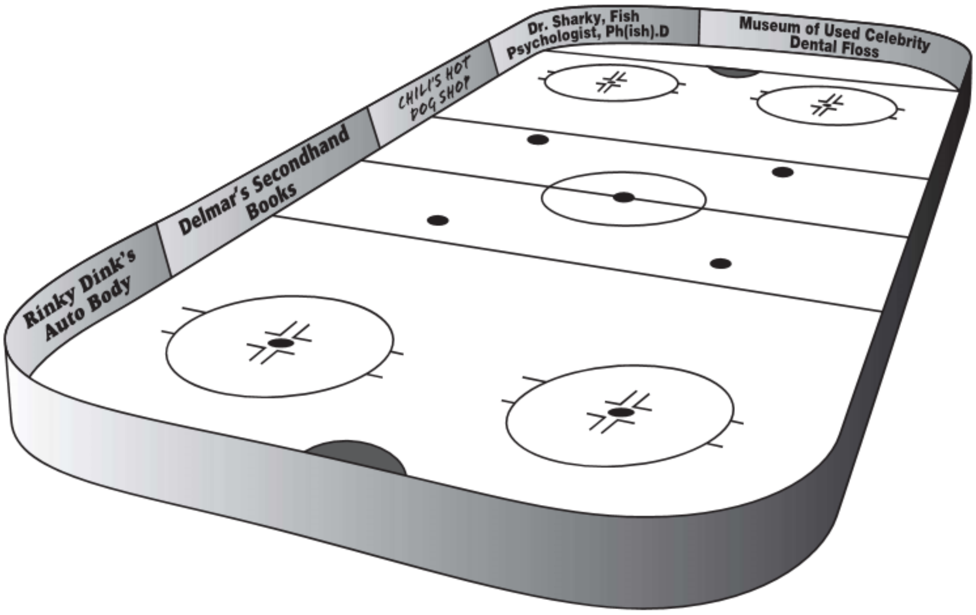
PROBLEM 1 This Is Quite a Work-Out

A work problem is a type of problem that involves the rates of several workers and the time it takes to complete a job. For example, the rate at which two painters work and the total time it takes them to paint a house while working together is an example of a work problem.

Anita and Martin are the assistant managers for the marketing department of the Snarky Larks Hockey Team. This hockey season is fast-approaching, and the rink board ads need to be mounted to the rink boards before the season begins.



Each ad is like a giant vinyl sticker that is stuck to each rink board along the inside of the hockey rink. It takes a team of three people to attach each ad: two people hold the ad while a third person carefully presses it to the rink board, being careful that it does not wrinkle.

Up until last year, Anita's team and Martin's team have taken turns doing this job—Anita's team attached the rink boards for the first season, Martin's team attached them for the next season, the following season Anita's team attached them, and so on.

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1. It takes Anita's team 20 hours to attach all of the rink boards, and it takes Martin's team 30 hours to attach all of the rink boards. This year, however, their boss has asked them to work together to get the job done faster.

   a. Determine the portion of the rink each team completes in the given number of hours.

   Anita's Team

   1 hour:

   5 hours:

   10 hours:

   Martin's Team

   1 hour:

   5 hours:

   10 hours:

   b. Consider the amount of the rink that each team can complete in x hours.

      i. Write an expression to represent the portion of the rink that Anita's team can complete in x hours.

      ii. Write an expression that represents the portion of the rink that Martin's team can complete in x hours.

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c. Each team's rate of work is defined as number of jobs completed per hour. In this case, the rate of work is the number of rinks completed per hour.

i. Determine Anita's team's rate of work.

ii. Determine Martin's team's rate of work.

d. Complete the table.

	Portion of the Rink Completed	Time Spent Working	Rate of Work
	Rinks	Hours	$\frac{\text{Rinks}}{\text{Hour}}$
Anita's Team		x	
Martin's Team		x	
Entire Job, or 1 Rink		x	

e. Consider the expression from the table that represents the portion of the rink that Anita and Martin's teams can complete when working together. If you want to determine the total time it takes the two teams to complete one rink while working together, what should you set this expression equal to?

f. Write and solve an equation to determine the total time it takes the two teams to complete the rink.

g. Suppose that the two teams work together attaching rink board ads for 4 hours each day. How many days will it take them to complete the job?

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Maureen is a community volunteer. She volunteers by watering the large vegetable garden in her neighborhood. Sometimes, Maureen's friend Sandra also volunteers.

2. It takes Maureen 90 minutes to water the garden. When Maureen and Sandra are working together, they can complete the job in 40 minutes.

   a. Complete the table. Let *x* represent the time it takes Sandra to water the garden if she works alone.

	Portion of the Garden Watered	Time Spent Watering	Rate of Watering
	Gardens	Minutes	$\frac{\text{Gardens}}{\text{Minute}}$
Maureen		40	
Sandra		40	
Entire Job, or 1 Garden		40	

   b. Write and solve an equation to determine the total time it takes Sandra to water the garden if she were working alone.

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PROBLEM 2 Shhh! The Mixmaster Needs Complete Concentration

A mixture problem is a type of problem that involves the combination of two or more liquids and the concentrations of those liquids.

1. Manuel is a taking a college chemistry course, and some of his time is spent in the chemistry lab. He is conducting an experiment for which he needs a 2% salt solution. However, all he can find in the lab is 120 milliliters (mL) of 10% salt solution.

   a. How many milliliters of salt and how many milliliters of water are in 120 mL of 10% salt solution?

   b. What would the concentration of the salt solution be if Manuel added 80 mL of water? 180 mL of water?

   c. Write and solve an equation to calculate the amount of water Manuel needs to add to the 120 mL of 10% salt solution to make a 2% salt solution. Let x represent the amount of water Manuel needs to add.

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2. Keisha is working on a chemistry experiment. She has 20 mL of a 20% sulfuric acid solution that she is mixing with a 5% sulfuric acid solution.

a. Describe the range of possible concentrations for the new solution.

b. Suppose that the 20 mL of 20% sulfuric acid solution is mixed with 10 mL of the 5% sulfuric acid solution. What is the concentration of the resulting solution? Explain your reasoning.

c. Write and solve an equation to calculate the amount of 5% sulfuric acid solution Keisha added if the resulting solution is a 12% sulfuric acid solution. Let *x* represent the amount of 5% sulfuric acid that Keisha added.

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PROBLEM 3 Are We There Yet?

A distance problem is a type of problem that involves distance, rate, and time.

1. A river barge travels 140 miles from a loading dock to a warehouse to deliver supplies. Then the barge returns to the loading dock. The barge travels with the current to the warehouse and against the current from the warehouse. The barge's total travel time is 20 hours, and it travels in still water at an average speed of 15 miles per hour.

   a. Use the given information to complete the table. Let x represent the average speed of the current.

	Distance Traveled	Time Traveled	Average Speed
	Miles	Hours	$\frac{\text{Miles}}{\text{Hours}}$
With the Current			$15 + x$
Against the Current	140		
Round Trip		20	

   b. You are given that the barge's total travel time is 20 hours. Write an algebraic expression that is equivalent to 20 hours.

   c. Write and solve an equation to calculate the average speed of the current.

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2. Calculate each value.

a. What is the barge's average speed during its trip to the warehouse?

b. What is the barge's average speed during its trip back to the loading dock?

c. How long does it take the barge to get from the loading dock to the warehouse?

d. How long does it take the barge to return to the loading dock from the warehouse?

e. Use your answers to parts (a) and (b) to calculate the average speed of the barge for the entire trip. Verify that your answer matches the table in Question 1.

f. Use your answers to parts (c) and (d) to calculate the barge's total travel time. Verify that your answer matches the table in Question 1.

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PROBLEM 4 How Much Is It?

A cost problem is a type of problem that involves the cost of ownership of an item over time.

Melinda has decided that it is time to replace her old refrigerator. She purchases a new Energy Star certified refrigerator. Energy Star certified refrigerators use less electricity than those which are not certified. In the long run, the Energy Star refrigerator should cost Melinda less to operate.

1. Melinda purchases a new Energy Star refrigerator for $2000. The refrigerator costs $46 per year to operate.

   a. Assume that the refrigerator is reliable and its only costs of ownership are the original cost and the cost of electricity. Determine Melinda's average annual cost of owning the new refrigerator in the given number of years.

   1 year:

   5 years:

   10 years:

   b. Write an expression to represent Melinda's average annual cost of owning the new refrigerator in x years.

   c. When Melinda's average annual cost of owning the refrigerator is less than $400, she plans to shop for a new television. When can Melinda shop for a new television?

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2. Melinda is curious to know how much money the Energy Star certified refrigerator will save her, compared to one that is not certified. A comparable non-certified model costs $1900 to purchase and is $60 per year to operate.

   a. Assume that this non-certified refrigerator's only costs of ownership are the original cost and the cost of electricity. Determine the average annual cost of owning this refrigerator in the given number of years.

      1 year:

      5 years:

      10 years:

   b. Write an expression to represent the average annual cost of owning the non-certified refrigerator in *x* years.

3. In how many years will the average annual cost of owning the Energy Star certified refrigerator be less than the average annual cost of owning the non-certified refrigerator? Show all of your work.

Be prepared to share your solutions and methods.